

Section 3.6 Matrices

When we are faced with solving a system of equations such as:
$$\begin{cases} 5x - 4y = -1 \\ -2x + 3y = 6 \end{cases}$$

We end up re-writing the x , y , and equal sign over and over.

A set up called Matrices eliminates this repetition. Matrices are used in most scientific calculators to solve systems of equations. All matrices really do is organize you and lead you directly to a solution. Without matrices, you could end up going in a circle and never get to solution. This is especially possible when doing 3 equations – 3 unknowns.

The order the system is written is vital: X values under x values, y values under y values, equal signs lined up and lastly the term with neither x nor y .

We use “square braces” to enclose a matrix. The elements of the matrix are only the coefficients or the constants. Thus:

$$\begin{bmatrix} 5 & -4 & -1 \\ -2 & 3 & 6 \end{bmatrix}$$

The objective of the matrix to start with $\begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}$ and arrive at a situation that looks

like this: $\begin{bmatrix} a & b & c \\ 0 & j & k \end{bmatrix}$. Then we take the j and the k out thus:
$$\begin{aligned} jy &= k \\ y &= \frac{k}{j} \end{aligned}$$

So for this equation I want to get a zero in position row 2, column 1.

We would proceed like we would do if we were doing the elimination method:

If I multiply the top row by 2 and the bottom row by 5 then add the top row to the bottom row, I will have the zero in row 2, column 1.

Doing this looks like:

$$\left| \begin{array}{l} 2 \begin{bmatrix} 5 & -4 & -1 \\ -2 & 3 & 6 \end{bmatrix} \\ \begin{bmatrix} 10 & -8 & -2 \\ -10 & 15 & 30 \end{bmatrix} \\ \begin{bmatrix} 10 & -8 & -2 \\ 0 & 7 & 28 \end{bmatrix} \end{array} \right.$$

The last line: $0 \ 7 \ 28$ means: $0x + 7y = 28$. So $y = 4$.

Substituting into one of the original equations leads us to $x = 3$. The conclusion of the problem is to write the coordinates: $(3, 4)$.

In the case of 3 X 3 (three equations and 3 unknowns)

we want to start with $\begin{bmatrix} a & b & c & d \\ e & f & g & h \\ i & j & k & l \end{bmatrix}$ and

arrive at a situation that looks like this: $\begin{bmatrix} a & b & c & d \\ 0 & m & n & p \\ 0 & 0 & q & r \end{bmatrix}$ where we have $qz = r$
 $z = \frac{r}{q}$.

We can switch rows in a matrix. Each row represents an equation. The order of the equations is really insignificant. I like to have a “one” coefficient for my row 1, column 1 item. That way I can multiply it by whatever I need so when I add that row to another, I will get the zero.

Basically what we are doing is multiplying an equality by some number (thus maintaining the equality) and adding that equality to another equality and its equality is maintained as well. The matrix method basically just focuses you to eliminate variables in a steady manner. This avoids going in circles. It is like having a compass on a hike.

Example 2

$$\begin{bmatrix} 2 & -1 & 4 & -3 \\ 1 & 0 & -4 & 5 \\ 6 & -1 & 2 & 10 \end{bmatrix}$$

Notice that the middle line has a 1 in the first position. I like to have that in position 1,1 so I will switch the first and second row.

$$\begin{bmatrix} 1 & 0 & -4 & 5 \\ 2 & -1 & 4 & -3 \\ 6 & -1 & 2 & 10 \end{bmatrix}$$

I will multiply the top row by -2 and add to the second row thus achieving the first step in my goal.

$$\begin{bmatrix} 1 & 0 & -4 & 5 \\ 0 & -1 & 12 & -13 \\ 6 & -1 & 2 & 10 \end{bmatrix}$$

The next zero I want is in row 3, column 1. I will multiply the top row by -6 and add to the bottom row. **Note that the row I multiply does *not* change.**

$$\begin{bmatrix} 1 & 0 & -4 & 5 \\ 0 & -1 & 12 & -13 \\ 0 & -1 & 26 & -20 \end{bmatrix}$$

I can get the last zero I require by multiplying the second line by -1 and adding that to the third row. Notice that because I have a zero in column 1 of the second line, I will **not** be changing the value in column 1 of the third row.

$$\begin{bmatrix} 1 & 0 & -4 & 5 \\ 0 & -1 & 12 & -13 \\ 0 & 0 & 14 & -7 \end{bmatrix}$$

$$14z = -7$$

The last line when removed from the matrix becomes $z = -\frac{1}{2}$.

$$-y - 6 = -13$$

I put *that* value in the second line getting: $-y = -7$.

$$y = 7$$

$$x + 0 \cdot 7 - 4\left(\frac{-1}{2}\right) = 5$$

I put both of those values into the first row getting: $x + 2 = 5$

$$x = 3$$

My final conclusion is: $\left(3, 7, -\frac{1}{2}\right)$

Next example.

I will put up each step of the problem. Your job will be to figure out what I did to get to that situation.

$$\text{The original problem is: } \begin{cases} 2x + y + 2z = 5 \\ 4x - 2y - 3z = 5 \\ -8x - y + z = -5 \end{cases}$$

$$\text{A) } \begin{bmatrix} 2 & 1 & 2 & 5 \\ 4 & -2 & -3 & 5 \\ -8 & -1 & 1 & -5 \end{bmatrix}$$

$$\text{B) } \begin{bmatrix} 2 & 1 & 2 & 5 \\ 0 & -4 & -7 & -5 \\ -8 & -1 & 1 & -5 \end{bmatrix}$$

$$\text{C) } \begin{bmatrix} 2 & 1 & 2 & 5 \\ 0 & -4 & -7 & -5 \\ 0 & 3 & 9 & 15 \end{bmatrix}$$

$$\text{D) } \begin{bmatrix} 2 & 1 & 2 & 5 \\ 0 & -12 & -21 & -15 \\ 0 & 3 & 9 & 15 \end{bmatrix}$$

$$\text{E) } \begin{bmatrix} 2 & 1 & 2 & 5 \\ 0 & -12 & -21 & -15 \\ 0 & 12 & 36 & 60 \end{bmatrix}$$

$$\text{F) } \begin{bmatrix} 2 & 1 & 2 & 5 \\ 0 & -12 & -21 & -15 \\ 0 & 0 & 15 & 45 \end{bmatrix}$$

$$\text{G) } \begin{aligned} &15z = 45 \\ &z = 3 \\ &\text{etc.} \end{aligned}$$

- A) Wrote matrix of coefficients from equations.
- B) Multiplied row 1 by -2 and added to second row.
- C) Multiplied row 1 by 4 and added to third row.
- D) Multiplied row 2 by 3.
- E) Multiplied row 3 by 4.
- F) Added row 2 to row 3.
- G) Removed last line from the matrix.

Section 3.8 Break Even Analysis and Supply and Demand.

The most difficult part of this section is learning the vocabulary.

Cost = There are usually two kinds of cost. Fixed cost are *not* related to the number of units of the product made such as rent for the building. Variable costs are costs that *are* related to the number of products made such as materials in the product.

Revenue = is sales price times the number sold.

Profit is Revenue minus Cost.

The “Break-even” point is when profit changes from negative (loss) to positive. This occurs when Revenue equals Cost.

We can find the intersection of the equation describing Revenue with the equation describing Cost as we have done earlier in this chapter.

Another set of words that sometimes cause confusion are Supply, Demand, and equilibrium point.

Supply refers to how much of an item is available for sale. When the price of an item is high, suppliers will get more of those items and purchasers will buy fewer of those available items.

Demand refers to how many much of an item is desired by purchasers. When the price of an item is low, purchasers will want more of those items and suppliers will not try to get more of those items.

When the amount of an item is available (Supply) is equal to the amount of an item desired (Demand) we are at an “equilibrium point”. The text has an excellent quote on page 210: **“The situation is analogous to a buyer and a seller negotiating the price of an item. The equilibrium point is the price and quantity that they finally agree on.”**

Copy the original problem. Write the resulting trinomial (usually) directly under the two binomials. Be very sure that your work is clear and legible. Write down your beginning time, your ending time and figure out your elapsed time for each assignment. Write the elapsed time on your Homework.

1. $(5X - 4)(7X - 3)$
2. $(4X - 2)(9X - 8)$
3. $(8X - 12)(9X + 5)$
4. $(9X + 10)(7X + 12)$
5. $(3X - 1)(5X - 10)$
6. $(7X - 7)(7X - 5)$
7. $(2X + 8)(8X - 7)$
8. $(7X - 5)(11X + 10)$
9. $(6X - 1)(6X - 4)$
10. $(2X - 8)(6X + 6)$
11. $(2X + 10)(7X - 3)$
12. $(7X + 8)(9X + 10)$
13. $(5X + 12)(3X - 3)$
14. $(11X + 6)(9X + 2)$
15. $(9X - 6)(8X - 7)$
16. $(4X - 8)(12X - 7)$
17. $(12X - 10)(8X + 5)$
18. $(5X - 3)(12X + 3)$
19. $(4X + 7)(10X + 6)$
20. $(4X - 2)(X + 3)$
21. $(8X - 12)(7X + 9)$
22. $(X + 10)(10X - 11)$
23. $(11X + 5)(2X - 10)$
24. $(7X - 11)(2X - 9)$
25. $(6X - 11)(5X + 8)$
26. $(X - 9)(12X + 7)$
27. $(4X - 6)(10X + 1)$
28. $(8X - 1)(8X - 4)$
29. $(6X + 7)(6X - 12)$
30. $(12X + 1)(4X - 8)$
31. $(2X + 7)(2X - 8)$
32. $(3X + 2)(7X + 7)$
33. $(10X - 6)(X + 3)$
34. $(6X - 10)(4X - 5)$
35. $(11X - 9)(X + 11)$
36. $(5X + 10)(7X + 5)$
37. $(4X - 2)(5X - 10)$
38. $(5X - 10)(7X - 5)$
39. $(11X - 9)(3X + 3)$
40. $(2X - 1)(2X - 10)$
41. $(8X - 11)(8X + 5)$
42. $(11X - 8)(2X + 11)$
43. $(2X + 12)(5X + 11)$
44. $(10X + 9)(11X + 5)$
45. $(7X - 8)(10X - 8)$
46. $(3X + 4)(8X + 8)$
47. $(X - 8)(7X - 5)$
48. $(10X + 9)(9X + 2)$
49. $(9X + 11)(12X + 12)$
50. $(10X - 1)(11X - 1)$
51. $(6X - 9)(3X - 4)$
52. $(2X + 4)(5X + 2)$
53. $(9X - 11)(11X - 4)$
54. $(6X - 10)(9X + 2)$
55. $(2X - 4)(3X - 7)$
56. $(4X - 6)(X + 4)$
57. $(8X + 11)(8X - 10)$
58. $(11X - 10)(12X - 9)$
59. $(6X - 12)(7X - 3)$
60. $(7X - 2)(2X - 9)$
61. $(6X + 4)(10X + 3)$
62. $(3X - 5)(6X - 1)$
63. $(6X + 6)(6X - 11)$
64. $(4X + 8)(X - 11)$
65. $(7X + 11)(6X + 10)$
66. $(11X + 1)(6X - 5)$
67. $(5X + 11)(2X - 3)$
68. $(11X + 2)(10X - 9)$
69. $(X - 5)(9X - 3)$
70. $(7X + 12)(4X - 5)$
71. $(9X + 7)(8X + 12)$
72. $(6X + 5)(3X - 9)$
73. $(6X - 8)(3X - 7)$
74. $(11X - 10)(6X + 2)$
75. $(5X - 3)(11X - 8)$
76. $(X + 10)(2X + 8)$
77. $(8X - 5)(7X - 2)$
78. $(11X - 8)(6X + 6)$
79. $(9X - 5)(3X + 8)$
80. $(5X + 11)(6X - 11)$
81. $(8X + 10)(2X - 9)$
82. $(9X - 8)(5X + 12)$
83. $(7X - 9)(7X + 9)$
84. $(10X - 7)(9X + 9)$
85. $(5X - 4)(7X + 12)$
86. $(10X + 9)(X + 1)$
87. $(6X + 7)(12X - 3)$
88. $(11X - 10)(8X + 9)$
89. $(X + 7)(2X + 7)$
90. $(11X - 5)(6X + 5)$
91. $(5X + 5)(8X - 11)$
92. $(3X - 8)(12X + 5)$
93. $(X + 1)(X + 4)$
94. $(6X + 7)(3X - 1)$
95. $(X + 5)(7X + 8)$
96. $(X + 7)(10X - 3)$
97. $(3X + 5)(9X - 2)$
98. $(8X - 7)(11X + 3)$
99. $(4X + 11)(2X + 10)$
100. $(8X + 5)(9X - 12)$
101. $(11X - 7)(5X + 2)$
102. $(12X + 2)(11X + 11)$
103. $(3X + 9)(X + 2)$
104. $(3X + 8)(7X + 1)$
105. $(9X - 3)(10X + 8)$
106. $(2X + 10)(2X - 2)$
107. $(5X + 1)(9X + 8)$
108. $(10X + 11)(8X + 9)$
109. $(10X - 1)(9X - 11)$
110. $(4X - 3)(11X - 12)$
111. $(11X + 7)(4X + 1)$
112. $(2X - 2)(7X - 2)$
113. $(5X - 4)(3X + 12)$
114. $(10X + 2)(X - 6)$
115. $(2X - 5)(X - 3)$
116. $(3X + 8)(8X - 6)$
117. $(2X + 11)(12X - 7)$
118. $(11X + 10)(12X + 9)$
119. $(6X + 2)(9X + 4)$
120. $(9X - 8)(8X - 3)$
121. $(7X - 9)(8X + 9)$
122. $(X + 4)(7X - 12)$
123. $(12X - 3)(12X + 1)$
124. $(10X + 2)(3X - 1)$
125. $(2X + 10)(7X - 12)$
126. $(2X - 11)(5X + 2)$
127. $(X - 4)(8X + 8)$
128. $(9X - 10)(12X - 8)$
129. $(11X - 11)(8X + 5)$
130. $(10X - 2)(12X - 1)$
131. $(9X + 12)(11X + 4)$
132. $(2X + 6)(6X - 5)$
133. $(10X + 9)(X - 3)$
134. $(11X + 8)(7X - 1)$
135. $(6X - 7)(12X - 7)$
136. $(3X + 6)(3X + 2)$
137. $(6X - 10)(9X + 12)$
138. $(9X - 4)(11X - 8)$
139. $(3X - 9)(4X + 9)$
140. $(9X - 11)(8X + 12)$
141. $(4X - 7)(9X - 9)$
142. $(X - 9)(3X - 8)$
143. $(11X - 2)(4X + 1)$
144. $(10X + 4)(4X - 10)$

The following are the answers to Foil-E. Check your answers *after doing the problems* to be certain you have ability to FOIL two binomials.

109. $90x^2 - 119x + 11$
110. $44x^2 - 81x + 36$
111. $44x^2 + 39x + 7$
112. $14x^2 - 18x + 4$
113. $15x^2 + 48x - 48$
114. $10x^2 - 58x - 12$
115. $2x^2 - 11x + 15$
116. $24x^2 + 46x - 48$
117. $24x^2 + 118x - 77$
118. $132x^2 + 219x + 90$
119. $54x^2 + 42x + 8$
120. $72x^2 - 91x + 24$
121. $56x^2 - 9x - 81$
122. $7x^2 + 16x - 48$
123. $144x^2 - 24x - 3$
124. $30x^2 - 4x - 2$
125. $14x^2 + 46x - 120$
126. $10x^2 - 51x - 22$
127. $8x^2 - 24x - 32$
128. $108x^2 - 192x + 80$
129. $88x^2 - 33x - 55$
130. $120x^2 - 34x + 2$
131. $99x^2 + 168x + 48$
132. $12x^2 + 26x - 30$
133. $10x^2 - 21x - 27$
134. $77x^2 + 45x - 8$
135. $72x^2 - 126x + 49$
136. $9x^2 + 24x + 12$
137. $54x^2 - 18x - 120$
138. $99x^2 - 116x + 32$
139. $12x^2 - 9x - 81$
140. $72x^2 + 20x - 132$
141. $36x^2 - 99x + 63$
142. $3x^2 - 35x + 72$
143. $44x^2 + 3x - 2$
144. $40x^2 - 84x - 40$

Copy the original problem. Write the resulting trinomial (usually) directly under the two binomials. Be very sure that your work is clear and legible. Write down your beginning time, your ending time and figure out your elapsed time for each assignment. Write the elapsed time on your Homework.

- | | | |
|------------------------|------------------------|-------------------------|
| 1. $(X - 6)(X + 4)$ | 49. $(2X - 2)(X + 2)$ | 97. $(5X - 1)(4X + 7)$ |
| 2. $(X - 8)(2X + 8)$ | 50. $(4X - 6)(X + 5)$ | 98. $(4X - 2)(2X + 7)$ |
| 3. $(X + 7)(4X - 5)$ | 51. $(2X + 8)(3X - 8)$ | 99. $(4X + 8)(2X - 6)$ |
| 4. $(4X - 6)(2X + 2)$ | 52. $(X - 8)(5X + 9)$ | 100. $(2X + 6)(X - 8)$ |
| 5. $(X - 7)(X - 9)$ | 53. $(5X + 3)(X - 2)$ | 101. $(4X + 5)(5X - 7)$ |
| 6. $(3X - 6)(3X + 2)$ | 54. $(4X - 4)(X - 1)$ | 102. $(3X - 9)(5X + 4)$ |
| 7. $(2X + 6)(3X - 6)$ | 55. $(X + 8)(4X - 2)$ | 103. $(X - 1)(4X + 4)$ |
| 8. $(4X - 3)(X + 8)$ | 56. $(3X + 4)(5X - 7)$ | 104. $(2X + 7)(X - 9)$ |
| 9. $(X + 2)(2X - 5)$ | 57. $(X - 7)(2X - 6)$ | 105. $(4X + 1)(X - 4)$ |
| 10. $(5X - 4)(5X - 1)$ | 58. $(X + 9)(X + 1)$ | 106. $(X + 3)(2X - 3)$ |
| 11. $(X + 2)(X + 6)$ | 59. $(4X - 3)(2X + 7)$ | 107. $(3X + 7)(4X - 4)$ |
| 12. $(3X - 4)(3X - 9)$ | 60. $(3X - 1)(3X + 3)$ | 108. $(2X + 8)(4X + 4)$ |
| 13. $(4X + 2)(4X - 7)$ | 61. $(X + 8)(2X - 6)$ | 109. $(X + 9)(3X - 4)$ |
| 14. $(3X - 3)(5X + 7)$ | 62. $(X - 8)(3X - 7)$ | 110. $(2X + 1)(4X + 8)$ |
| 15. $(5X + 8)(4X - 4)$ | 63. $(5X - 7)(3X + 8)$ | 111. $(3X + 9)(5X + 8)$ |
| 16. $(3X - 1)(3X - 8)$ | 64. $(X - 4)(5X - 7)$ | 112. $(2X - 1)(2X + 4)$ |
| 17. $(4X - 3)(4X - 7)$ | 65. $(X + 2)(2X + 1)$ | 113. $(3X + 4)(2X + 3)$ |
| 18. $(3X - 9)(X - 4)$ | 66. $(X + 5)(5X - 9)$ | 114. $(5X - 7)(2X + 9)$ |
| 19. $(2X - 1)(3X + 5)$ | 67. $(2X + 4)(2X - 7)$ | 115. $(5X - 3)(4X + 6)$ |
| 20. $(X - 6)(4X + 4)$ | 68. $(3X - 8)(4X + 5)$ | 116. $(5X - 1)(4X + 2)$ |
| 21. $(2X - 6)(2X - 9)$ | 69. $(5X + 5)(X + 2)$ | 117. $(5X + 4)(4X - 3)$ |
| 22. $(5X - 2)(5X - 2)$ | 70. $(5X - 3)(5X + 3)$ | 118. $(4X - 9)(5X - 1)$ |
| 23. $(2X - 5)(5X + 8)$ | 71. $(X - 8)(4X + 4)$ | 119. $(3X + 4)(4X - 5)$ |
| 24. $(3X - 5)(3X + 4)$ | 72. $(3X - 2)(3X - 1)$ | 120. $(4X - 6)(3X + 1)$ |
| 25. $(3X + 6)(4X - 5)$ | 73. $(X - 4)(4X + 8)$ | 121. $(3X + 5)(4X + 7)$ |
| 26. $(2X + 1)(4X - 3)$ | 74. $(2X - 2)(5X - 9)$ | 122. $(X - 4)(X + 5)$ |
| 27. $(3X + 6)(4X + 2)$ | 75. $(4X - 9)(5X + 5)$ | 123. $(3X - 6)(3X - 9)$ |
| 28. $(5X + 3)(5X - 3)$ | 76. $(4X + 4)(3X - 8)$ | 124. $(2X - 1)(4X + 6)$ |
| 29. $(X - 6)(3X - 1)$ | 77. $(5X + 7)(5X - 2)$ | 125. $(2X - 1)(X + 9)$ |
| 30. $(2X - 6)(4X + 8)$ | 78. $(3X + 5)(5X - 6)$ | 126. $(3X - 6)(X + 5)$ |
| 31. $(2X - 6)(4X + 8)$ | 79. $(5X - 4)(3X + 1)$ | 127. $(3X - 6)(4X + 5)$ |
| 32. $(3X - 3)(2X - 6)$ | 80. $(5X + 2)(3X + 9)$ | 128. $(5X + 4)(4X - 6)$ |
| 33. $(5X + 2)(4X + 2)$ | 81. $(4X - 5)(5X + 2)$ | 129. $(5X - 6)(3X - 3)$ |
| 34. $(X + 2)(2X - 6)$ | 82. $(X - 2)(3X - 1)$ | 130. $(5X - 7)(3X + 5)$ |
| 35. $(X + 7)(2X - 2)$ | 83. $(2X + 6)(5X - 2)$ | 131. $(3X + 6)(4X - 3)$ |
| 36. $(2X - 9)(5X - 9)$ | 84. $(5X - 5)(5X - 8)$ | 132. $(5X - 8)(3X + 4)$ |
| 37. $(X - 1)(5X + 8)$ | 85. $(3X - 4)(X - 8)$ | 133. $(2X - 9)(4X + 9)$ |
| 38. $(4X - 8)(4X - 9)$ | 86. $(2X + 5)(X - 4)$ | 134. $(5X - 3)(4X + 8)$ |
| 39. $(2X + 8)(3X + 8)$ | 87. $(4X - 9)(2X + 4)$ | 135. $(X - 3)(5X - 9)$ |
| 40. $(2X + 2)(3X + 7)$ | 88. $(4X - 3)(4X - 2)$ | 136. $(X - 2)(X + 2)$ |
| 41. $(X - 3)(5X + 2)$ | 89. $(2X - 8)(2X - 6)$ | 137. $(5X - 8)(X - 9)$ |
| 42. $(5X - 9)(2X + 2)$ | 90. $(X + 4)(2X + 4)$ | 138. $(X + 5)(4X + 5)$ |
| 43. $(2X - 3)(5X + 1)$ | 91. $(X + 3)(2X - 4)$ | 139. $(2X + 5)(X - 7)$ |
| 44. $(2X + 9)(3X - 6)$ | 92. $(5X - 2)(X - 8)$ | 140. $(X - 6)(4X - 2)$ |
| 45. $(4X - 1)(5X - 2)$ | 93. $(5X - 4)(2X - 6)$ | 141. $(X + 2)(4X + 2)$ |
| 46. $(4X - 2)(4X - 8)$ | 94. $(4X + 4)(4X + 4)$ | 142. $(2X + 6)(3X - 5)$ |
| 47. $(2X - 4)(4X + 8)$ | 95. $(X - 7)(2X + 1)$ | 143. $(4X - 2)(X + 3)$ |
| 48. $(4X + 7)(3X - 9)$ | 96. $(2X - 2)(4X + 6)$ | 144. $(3X - 4)(2X + 5)$ |

Sections 5.3 – 5.7 deal with factoring.

The text tries to divide factoring into various categories but this approach does not seem to be very effective.

We need to be able to identify the factoring situation at hand and know what to do with it.

Common Factors – we look for the largest number that is a factor of all numbers. When we factor a common factor from an expression **with 4 terms**, we will have the factor on the outside and we will have **exactly 4 terms** inside the parentheses. The most common error in this regard is:

$$2 + 4x^2 - 6xy + 8y^3$$

$$2(2x^2 - 3xy + 4y^3)$$

What is wrong with this? Original had 4 terms. We have 3 terms inside the parentheses.

Properly done, you should get:

$$2 + 4x^2 - 6xy + 8y^3$$

$$2(1 + 2x^2 - 3xy + 4y^3)$$

Remember that you should check your factoring after you have done it. You should be able to multiply (clear the parentheses) and get what you started with.

unFOIL – Make lists of *all* factors for the leading coefficient and for the constant. Make your selection from those lists trying to get the middle terms to work.

$$\begin{array}{ccc}
 1 \cdot 12 & & 1 \cdot 15 \\
 2 \cdot 6 & 12x^2 - 8x - 15 & 3 \cdot 5 \\
 3 \cdot 4 & &
 \end{array}$$

Because the constant is negative, the signs will be different.

Had the constant been positive, the signs would have been the same as the sign of the middle term.

Sub cases in unFOIL include “difference of squares” and “perfect squares”. (Although one wonders what an “imperfect” square might be.) Both of these have important uses throughout algebra and we will see them before we finish the semester.

Sum/Difference of Cubes – This is another area where factoring is nearly automatic.

I use this formula for the Sum/Difference of Cubes: $(a^3 \pm b^3) = (a \pm b)(a^2 \mp ab + b^2)$ and

I translate it into English as:

A first number cubed plus or minus a second number cubed equals

“first agree second times first squared disagree product of the two always a plus second number squared.”

As was the case when we were working with clearing parentheses, practice is the only thing that makes this really easy. And, I should point out, that the practice must be done very quickly. You need to work fast, to force those combinations into your mind and to allow yourself to recognize the various situations.

Remember: No equal signs!

Copy the original problem. If there is a common factor in the trinomial, factor it out. Factor the resulting trinomial (usually) into the two binomials. Be very sure that your work is clear and legible.

1. $12X^2 - X - 20$
2. $5X^2 + 7X - 6$
3. $15X^2 + 13X + 2$
4. $3X^2 + 9X - 12$
5. $18X^2 - 12X - 6$
6. $6X^2 - 11X + 3$
7. $8X^2 - 12X + 4$
8. $10X^2 + 17X + 3$
9. $2X^2 + 5X + 3$
10. $6X^2 - 14X + 8$
11. $18X^2 + 9X - 2$
12. $10X^2 + 3X - 18$
13. $X^2 + 3X - 4$
14. $9X^2 - 3X - 12$
15. $10X^2 - 13X + 4$
16. $15X^2 + 3X - 12$
17. $8X^2 - 8X - 6$
18. $3X^2 + 6X - 9$
19. $X^2 + 3X - 18$
20. $6X^2 - 22X + 12$
21. $6X^2 + 11X + 4$
22. $9X^2 - 12X - 12$
23. $6X^2 - 18X + 12$
24. $6X^2 + 8X - 8$
25. $12X^2 + 19X + 4$
26. $2X^2 - 6X + 4$
27. $8X^2 + 2X - 1$
28. $12X^2 - 28X + 16$
29. $6X^2 + 5X + 1$
30. $6X^2 - 6$
31. $6X^2 - 21X + 9$
32. $3X^2 + 2X - 1$
33. $9X^2 + 30X + 24$
34. $2X^2 + 6X - 8$
35. $5X^2 + 16X + 3$
36. $5X^2 + 4X - 1$
37. $5X^2 + 14X - 24$
38. $3X^2 - 2X - 8$
39. $9X^2 + 6X - 3$
40. $X^2 + 2X - 8$
41. $6X^2 - 13X - 15$
42. $18X^2 - 2$
43. $2X^2 - 8X - 10$
44. $10X^2 + 23X + 12$
45. $8X^2 - 2X - 10$
46. $6X^2 - 25X + 4$
47. $10X^2 - 3X - 1$
48. $2X^2 - 9X - 18$
49. $6X^2 + 9X + 3$
50. $3X^2 + 6X + 3$
51. $18X^2 - 24X + 8$
52. $12X^2 + 6X - 18$
53. $8X^2 - 20X + 12$
54. $3X^2 + 10X + 3$
55. $18X^2 + 3X - 3$
56. $3X^2 + 19X + 20$
57. $6X^2 + 15X + 9$
58. $4X^2 - 7X - 2$
59. $2X^2 + 4X - 16$
60. $10X^2 + X - 2$
61. $X^2 - 2X - 15$
62. $6X^2 + 11X - 10$
63. $X^2 + 4X + 3$
64. $12X^2 - 14X + 4$
65. $6X^2 - 7X - 5$
66. $3X^2 - 15X + 18$
67. $9X^2 + 18X + 9$
68. $12X^2 + 22X + 10$
69. $3X^2 + 6X + 3$
70. $6X^2 + 29X + 20$
71. $2X^2 - 14X + 24$
72. $10X^2 - 5X - 5$
73. $9X^2 + 18X + 9$
74. $4X^2 - 2X - 12$
75. $2X^2 - 6X - 8$
76. $3X^2 - 9X + 6$
77. $6X^2 + 4X - 16$
78. $X^2 + 7X + 6$
79. $15X^2 + 8X - 16$
80. $6X^2 + 8X + 2$
81. $5X^2 + 13X - 6$
82. $2X^2 - 4X - 6$
83. $2X^2 - 8X + 6$
84. $15X^2 - 2X - 24$
85. $18X^2 + 12X - 6$
86. $3X^2 + 9X + 6$
87. $6X^2 - 16X + 10$
88. $X^2 - 5X + 4$
89. $10X^2 - 4X - 6$
90. $6X^2 - 13X + 6$
91. $5X^2 - 23X + 12$
92. $4X^2 + 6X + 2$
93. $3X^2 + 3X - 6$
94. $3X^2 + 17X + 20$
95. $4X^2 + 22X + 24$
96. $4X^2 - 4$
97. $9X^2 + 12X - 12$
98. $2X^2 - 10X + 8$
99. $3X^2 + 2X - 1$
100. $10X^2 + 8X - 24$
101. $10X^2 - 19X + 6$
102. $X^2 - 2X - 3$
103. $6X^2 - 3X - 18$
104. $8X^2 - 6X - 5$
105. $18X^2 - 42X + 24$
106. $2X^2 + 4X - 6$
107. $12X^2 - 17X + 6$
108. $10X^2 + 21X + 9$
109. $X^2 + 6X + 5$
110. $9X^2 - 3X - 6$
111. $4X^2 + 16X + 16$
112. $12X^2 + 10X - 12$
113. $3X^2 - 8X - 3$
114. $3X^2 - 4X - 4$
115. $8X^2 + 10X + 3$
116. $2X^2 + 6X + 4$
117. $3X^2 - 4X - 15$
118. $5X^2 + 9X - 2$
119. $18X^2 - 6X - 12$
120. $5X^2 - 16X + 3$
121. $X^2 - 10X + 24$
122. $10X^2 - 14X + 4$
123. $18X^2 + 36X + 16$
124. $10X^2 - 3X - 4$
125. $6X^2 + 30X + 24$
126. $5X^2 - 6X + 1$
127. $3X^2 - 5X - 2$
128. $18X^2 + 27X + 9$
129. $12X^2 - 2X - 2$
130. $10X^2 + 30X + 20$
131. $10X^2 + 8X - 24$
132. $6X^2 - 6$
133. $5X^2 + 9X - 18$
134. $12X^2 + 2X - 4$
135. $8X^2 + 4X - 12$
136. $2X^2 - 8$
137. $8X^2 + 14X + 5$
138. $10X^2 - 7X - 12$
139. $8X^2 + 2X - 15$
140. $9X^2 - 3X - 12$
141. $X^2 + 4X - 5$
142. $X^2 + 9X + 20$
143. $X^2 - 6X + 8$
144. $6X^2 - 14X + 4$

These are the solutions to
Tri-L.

62. $(3X - 2)(2X + 5)$
63. $(X + 3)(X + 1)$
64. $2(2X - 1)(3X - 2)$
65. $(2X + 1)(3X - 5)$
66. $3(X - 3)(X - 2)$
67. $9(X + 1)(X + 1)$
68. $2(X + 1)(6X + 5)$
69. $3(X + 1)(X + 1)$
70. $(X + 4)(6X + 5)$
71. $2(X - 4)(X - 3)$
72. $5(2X + 1)(X - 1)$
73. $(3X + 3)(3X + 3)$
74. $2(X - 2)(2X + 3)$
75. $2(X - 4)(X + 1)$
76. $3(X - 2)(X - 1)$
77. $(2X + 4)(3X - 4)$
78. $(X + 1)(X + 6)$
79. $(3X + 4)(5X - 4)$
80. $2(3X + 1)(X + 1)$

These are the solutions to
Tri-D

41. $(2X + 5)(2X - 9)$
42. $(3X - 7)^2$
43. $4(5X + 4)(5X - 3)$
44. $12(X + 1)^2$
45. $6(2X - 1)(6X + 11)$
46. $9(X - 1)(X - 5)$
47. $(7X + 5)(3X + 4)$
48. $(5X - 11)(X - 10)$
49. $2(5X + 7)(2X - 3)$
50. $3(7X - 10)(3X - 4)$
51. $(4X + 1)(3X + 4)$
52. $(X - 7)(X - 5)$
53. $(12X + 7)(7X + 8)$
54. $(X - 9)(5X - 9)$
55. $(4X + 9)(3X - 8)$
56. $2(3X + 2)(4X - 11)$
57. $10(X + 1)(X - 6)$
58. $2(3X - 1)(3X + 4)$
59. $(4X - 9)(11X - 3)$
60. $(6X + 11)(7X + 8)$

Copy the original problem. If there is a common factor in the trinomial, factor it out. Factor the resulting trinomial (usually) into the two binomials. Be very sure that your work is clear and legible.

- | | | |
|----------------------|----------------------|-----------------------|
| 1. $10X^2-117X-36$ | 49. $20X^2-2X-42$ | 97. $54X^2+144X+90$ |
| 2. $15X^2+15X-30$ | 50. $63X^2-174X+120$ | 98. $44X^2-26X+2$ |
| 3. $X^2+13X+42$ | 51. $12X^2+19X+4$ | 99. $30X^2-93X+66$ |
| 4. $21X^2-11X-40$ | 52. $X^2-12X+35$ | 100. $11X^2-50X-25$ |
| 5. $36X^2+31X-56$ | 53. $84X^2+145X+56$ | 101. $8X^2+33X+4$ |
| 6. $77X^2+108X+36$ | 54. $5X^2-54X+81$ | 102. $120X^2+26X-77$ |
| 7. $X^2-17X+72$ | 55. $12X^2-5X-72$ | 103. $90X^2+46X+4$ |
| 8. $15X^2-2X-77$ | 56. $24X^2-50X-44$ | 104. $88X^2+49X-99$ |
| 9. $21X^2-11X-2$ | 57. $10X^2-50X-60$ | 105. $14X^2+47X-7$ |
| 10. $50X^2-40X-120$ | 58. $18X^2+18X-8$ | 106. $12X^2-94X+132$ |
| 11. $10X^2-4X-32$ | 59. $44X^2-111X+27$ | 107. $22X^2+144X+72$ |
| 12. $10X^2+19X+6$ | 60. $42X^2+125X+88$ | 108. $30X^2-129X+36$ |
| 13. $21X^2+20X-25$ | 61. $12X^2+101X+40$ | 109. $30X^2-88X-40$ |
| 14. $100X^2+130X+30$ | 62. $60X^2+129X+63$ | 110. $33X^2+89X+28$ |
| 15. $55X^2-150X+80$ | 63. $8X^2+62X+42$ | 111. $8X^2+23X-3$ |
| 16. $60X^2-126X+12$ | 64. $99X^2+26X-45$ | 112. $24X^2-70X+44$ |
| 17. $18X^2-66X-120$ | 65. $42X^2-89X+22$ | 113. $10X^2+X-3$ |
| 18. $27X^2-51X-28$ | 66. $6X^2-6$ | 114. $9X^2+6X-15$ |
| 19. $77X^2-34X-16$ | 67. $10X^2-99X-10$ | 115. $90X^2+101X-11$ |
| 20. $48X^2+36X-30$ | 68. $6X^2+17X+12$ | 116. $49X^2+147X+108$ |
| 21. $22X^2-X-5$ | 69. $6X^2+3X-30$ | 117. $8X^2+44X+48$ |
| 22. $60X^2+64X-7$ | 70. $132X^2+125X+28$ | 118. $108X^2+90X+18$ |
| 23. $6X^2+40X+24$ | 71. $30X^2-64X+18$ | 119. $50X^2+90X-20$ |
| 24. $9X^2-59X-28$ | 72. $24X^2-92X+40$ | 120. $64X^2+8X-90$ |
| 25. $35X^2+6X-77$ | 73. $10X^2+80X+70$ | 121. $50X^2-30X+4$ |
| 26. $60X^2-148X+80$ | 74. $42X^2+2X-20$ | 122. $56X^2+86X+20$ |
| 27. $48X^2+40X+7$ | 75. $88X^2+217X+132$ | 123. $18X^2+31X+11$ |
| 28. $72X^2-156X+84$ | 76. $6X^2+4X-2$ | 124. $66X^2+38X-4$ |
| 29. $8X^2-32$ | 77. $14X^2-45X-14$ | 125. $28X^2-37X-11$ |
| 30. $10X^2+28X-110$ | 78. $108X^2-165X+50$ | 126. $18X^2+60X+48$ |
| 31. $42X^2-61X+14$ | 79. $8X^2-39X-54$ | 127. $8X^2-102X+72$ |
| 32. $44X^2-101X+42$ | 80. $64X^2-64X-9$ | 128. $5X^2-6X+1$ |
| 33. $10X^2+15X-10$ | 81. $66X^2+14X-20$ | 129. $36X^2+105X-99$ |
| 34. $16X^2-32X-20$ | 82. $12X^2-94X+132$ | 130. $81X^2+9X-72$ |
| 35. $10X^2+52X+10$ | 83. $84X^2+48X-132$ | 131. $64X^2+40X-24$ |
| 36. $88X^2+15X-25$ | 84. $5X^2+56X+60$ | 132. $54X^2+15X-21$ |
| 37. $3X^2-29X-44$ | 85. $27X^2+45X+12$ | 133. $63X^2-139X+70$ |
| 38. $20X^2-48X-77$ | 86. $14X^2-91X+77$ | 134. $132X^2+89X+7$ |
| 39. $72X^2+24X-6$ | 87. $48X^2-20X-42$ | 135. $81X^2-117X+42$ |
| 40. $48X^2+168X+72$ | 88. $132X^2+171X+54$ | 136. $6X^2-28X+16$ |
| 41. $4X^2-8X-45$ | 89. $45X^2-49X-110$ | 137. $21X^2+21X-42$ |
| 42. $9X^2-42X+49$ | 90. $48X^2+156X+90$ | 138. $24X^2-112X+120$ |
| 43. $100X^2+20X-48$ | 91. $10X^2-104X-66$ | 139. $40X^2-106X+36$ |
| 44. $12X^2+24X+12$ | 92. $40X^2+14X-45$ | 140. $24X^2-96$ |
| 45. $72X^2+96X-66$ | 93. $96X^2-100X+24$ | 141. $24X^2+88X+14$ |
| 46. $9X^2-54X+45$ | 94. $25X^2-95X+90$ | 142. $18X^2+20X+2$ |
| 47. $21X^2+43X+20$ | 95. $84X^2-33X-72$ | 143. $110X^2-23X-70$ |
| 48. $5X^2-61X+110$ | 96. $24X^2+30X-36$ | 144. $44X^2-97X-66$ |